$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \frac{x^n}{n!} + \cdots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

1. Find the first 4 terms of the series 2. Write the rule for the series 3. Find the interval of convergence 4. Take the derivative of the series 5. Take the anti-derivative of the series	5) $f(x) = x^2 e^{x^3}$ $$\sum_{n=0}^{\infty} \frac{x^2 (x^3)^n}{n!} = \sum_{n=0}^{\infty} \frac{x^{3n+2}}{n!} = x^2 + x^5 + \frac{x^8}{2} + \frac{x^{11}}{6}$$ $$\frac{d}{dx} \sum_{n=0}^{\infty} \frac{x^{3n+2}}{n!} = \sum_{n=0}^{\infty} \frac{(3n+2)x^{3n+1}}{n!}$$ $$\int \sum_{n=0}^{\infty} \frac{x^{3n+2}}{n!} = \sum_{n=0}^{\infty} \frac{x^{3n+3}}{(3n+3)n!} + C$$ 6) $f(x) = xe^{x^4}$

$$\tan^{-1}(x) = \boxed{\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}} = x - \frac{x^3}{3} + \frac{x^5}{5}$$

1. Find the first 4 terms of the series 2. Write the rule for the series 3. Find the interval of convergence 4. Take the derivative of the series 5. Take the anti-derivative of the series	7) $f(x)=x^2\tan^{-1}(x^5)$ $= \sum_{n=0}^{\infty} \frac{(-1)^n x^2 (x^5)^{2n+1}}{2n+1}$ $= \sum_{n=0}^{\infty} \frac{(-1)^n x^{10n+7}}{2n+1} = x^7 - \frac{x^{17}}{3} + \frac{x^{27}}{5} - \frac{x^{37}}{7}$ $\frac{d}{dx} \sum_{n=0}^{\infty} \frac{(-1)^n x^{10n+7}}{2n+1} = \sum_{n=0}^{\infty} \frac{(-1)^n (10n+7) x^{10n+6}}{2n+1}$ $\int \sum_{n=0}^{\infty} \frac{(-1)^n x^{10n+7}}{2n+1} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{10n+8}}{(10n+8)(2n+1)} + C$ 8) $f(x)=\ln(1-x^4)$

	Find the 3rd order Taylor Polynomial centered at x = 2
① Take 3 derivatives ② Plug in the center ③ Built Polynomial $\frac{f^n(c)(x-c)^n}{n!}$ where c = center	18) $f(x)=\frac{1}{x}$ $f(x)=x^{-1}=\frac{1}{x}$ $f(2)=1/2$ $f'(x)=-x^{-2}=-\frac{1}{x^2}$ $f'(2)=-1/4$ $f''(x)=2x^{-3}=2/x^3$ $f''(2)=2/8=1/4$ $f'''(x)=-6x^{-4}=-6/x^4$ $f'''(2)=-6/16=-3/8$ $P_3(x-2)=\frac{\frac{1}{2}(x-2)^0}{0!}-\frac{\frac{1}{4}(x-2)^1}{1!}+\frac{\frac{1}{4}(x-2)^2}{2!}-\frac{\frac{3}{8}(x-2)^3}{3!}$ $=\frac{1}{2}-\frac{1}{4}(x-2)+\frac{1}{8}(x-2)^2-\frac{1}{16}(x-2)^3$
$f(x)=\sin x$ $f'(x)=\cos x$ $f''(x)=-\sin x$ $f'''(x)=-\cos x$	19) $f(x)=\sin x$ at $x=\frac{\pi}{4}$ $f(\frac{\pi}{4})=\sqrt{2}/2$ $f'(\frac{\pi}{4})=\sqrt{2}/2$ $f''(\frac{\pi}{4})=-\sqrt{2}/2$ $f'''(\frac{\pi}{4})=-\sqrt{2}/2$ $P_3(x-\frac{\pi}{4})=\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}(x-\frac{\pi}{4})-\frac{\frac{\sqrt{2}}{2}(x-\frac{\pi}{4})^2}{2!}-\frac{\frac{\sqrt{2}}{2}(x-\frac{\pi}{4})^3}{3!}$

$-\frac{\sqrt{2}}{4}(x-\frac{\pi}{4})^2-\frac{\sqrt{2}}{12}(x-\frac{\pi}{4})^3$

	2015 BC6 1. The Maclaurin series for a function f is given by $\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{n} x^n = x - \frac{3}{2}x^2 + 3x^3 - \cdots + \frac{(-3)^{n-1}}{n} x^n \cdots$ and converges to f(x) for $\|x\| < R$, where R is the radius of convergence of the Maclaurin series. a) Use the Ratio Test to find R b) Write the first four non-zero terms of the Maclaurin series for f', the derivative of f. Express f' as a rational function for $\|x\| < R$. c) Write the first four nonzero terms of the Maclaurin series for e^x. Use the Maclaurin series for e^x to write the third-degree polynomial for $g(x) = e^x f(x)$ about x = 0.

	Let f b a function that has derivatives of all orders for all real numbers. Assume f(0) = 4, $f'(0)=5$, $f''(0)=-8$, and $f'''(0)=6$. a. Write the third order Taylor Polynomial for f at x = 0 and use it to approximate f(.2). b. Write the second order Taylor polynomial for f', at x = 0 c. Write the fourth order Taylor polynomial for $\int_0^x f(t)dt$, at x = 0. d. Determine if the linearization of f is an underestimate or overestimate near 0. p. 527 57 a. Write the first three nonzero terms and the general term of the Taylor Series generated by $f(x)=5\sin\left(\frac{x}{2}\right)$ at x = 0. c. What is the minimum number of terms of the series in part a needed to approximate f(x) on the interval (-2, 2) with an error not exceeding .1 in magnitude. Explain your answer.

	p. 492 #24 The Maclaurin Series for f(x) is $f(x)=1+\frac{x}{2!}+\frac{x^2}{3!}+\frac{x^3}{4!}+\cdots\frac{x^n}{(n+1)!}$. a. Find $f'(0)$ and $f^{10}(0)$. b. Let g(x) = xf(x). Write the Maclaurin Series for g(x), showing the first three non-zero terms and the general term. c. Write g(x) in terms of a familiar function without using series.

p. 500 #13

Find a formula for the truncation error if we use $P_6(x)$ to approximate $\frac{1}{1+2x}$ on (-.5, .5).

p. 500 20

a. If cos(x) is replaced by $1-\frac{x^2}{2}$ and $|x|<.5$, what estimate can be made of the error?

b. Does $1-\frac{x^2}{2}$ tend to be to large or to small.

p. 500 #22

The approximation $\sqrt{1+x}\approx 1+\frac{x}{2}$ is used when x is small. Estimate the error when $|x|<.1$

p. 527 #60

Let $f(x)=\dfrac{1}{x-2}$ at x = 3.

a. Write the first 4 terms and the general term of the Taylor Series generated by f(x) at x = 3.

b. Use the result in part (a) to find the fourth order polynomial and the general term of the series generated by $\ln|x-2|$ at x = 3.

c. Use the series in part (b) to compute a number that differs from ln(1.5) by less than 0.05. Justify your answer.

83. The Taylor Series for lnx, centered at x = 1, is $\sum_{n=1}^{\infty}\dfrac{(-1)^{n+1}(x-1)^n}{n}$. Let f be the function given by the sum of the first three nonzero terms of this series. The maximum value of $|\ln x - f(x)|$ for $.3 \le x \le 1.7$ is

(A) .030 (B) .039 (C) .145 (D) .153 (E) .529

2011 BC6

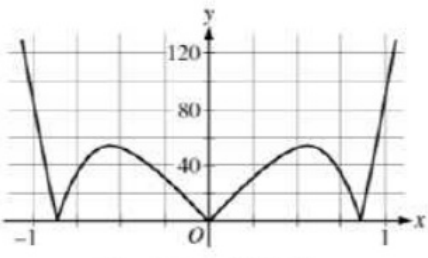
Let $f(x) = \sin(x^2) + \cos x$.

a. Write the first four nonzero terms of the Taylor series for sinx about x = 0, and write the first four nonzero terms of the Taylor series for $\sin(x^2)$ about x = 0.

b. Write the first four nonzero terms of the Taylor series for cosx about x = 0. Use this series and the series for $\sin(x^2)$, found in part a, to write the first four nonzero terms of the Taylor series for f(x) about x = 0.

c. Find the value of $f^{(6)}(0)$.

d. Let $f(x) = \sin(x^2) + \cos x$. The graph of $y = \left|f^{(5)}(x)\right|$ is shown.



Graph of $y = \left|f^{(5)}(x)\right|$

Let $P_4(x)$ be the fourth degree Taylor polynomial for f about x = 0. Using information from the graph of $y = \left|f^{(5)}(x)\right|$, shown above, show that $\left|P_4\left(\frac{1}{4}\right) - f\left(\frac{1}{4}\right)\right| < \frac{1}{3000}$.

2004 BC6

Let f be the function given by $f(x)=\sin\left(5x+\frac{\pi}{4}\right)$, and let P(x) be the third-degree Taylor polynomial for f about x = 0.

a) Find P(x).

b) Find the coefficient of x^{22} in the Taylor series about x = 0.

c) Use the Lagrange error bound to show that $\left|f\left(\frac{1}{10}\right)-P\left(\frac{1}{10}\right)\right|<\frac{1}{100}$.

d) Let G be the function given $G(x)=\int_0^x f(t)dt$. Write the third-degree Taylor polynomial for G about x = 0.

$f(x)=\sum_{n=0}^{\infty} c_n x^n$	INTERVAL OF CONVERGENCE	RADIUS OF CONVERGENCE
$\frac{1}{1-x}=\sum_{n=0}^{\infty} x^n = 1+x+x^2+x^3+\cdots$	$(-1,1)$	1
$e^x=\sum_{n=0}^{\infty}\frac{x^n}{n!}=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots$	$(-\infty,\infty)$	∞
$\sin x=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!} = x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\cdots$	$(-\infty,\infty)$	∞
$\cos x=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!} = 1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\cdots$	$(-\infty,\infty)$	∞
$\tan^{-1} x=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{2n+1} = x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\cdots$	$(-1,1]$	1
$\ln(1+x)=\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n} = x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots$	$(-1,1]$	1